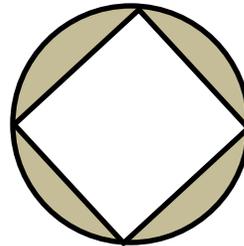


2014 IMSA Junior High Mathematics Competition

7<sup>th</sup> Grade Individual

1. Find the greatest common factor (GCF) of 72 and 64.
2. What is the value of  $(798^2 - 543^2) + (122^2 - 25)$ ?
3. Sarah rolls two distinct six-sided dice. What is the probability that both dice will show the same number? Express your answer as a common fraction.
4. Abby, Billy, Carrie, and Dill need to line up. If Abbie is not last, Billy comes right before Carrie, and Dill is before Abbie, who is 3<sup>rd</sup> in line?
5. How many lines of symmetry does a square have in the plane?
6. What is the perimeter of a right triangle with a 34 inch hypotenuse and a 1 foot leg? Write your answer in inches in simplest form.
7. Malachi is deciding to eat healthy. A salad bar has seven sections, each with 2 choices. If Malachi decides to choose one food from each section, how many different kinds of salads can he make?
8. The circumscribed circle of a square is shown. The radius of the circle is 6 in. What is the area of the shaded region of this figure?



9. How many different sequences of five letters can be made from H, I, J, K, and L if each letter may be used more than once and if the sequence may not start or end with a vowel?

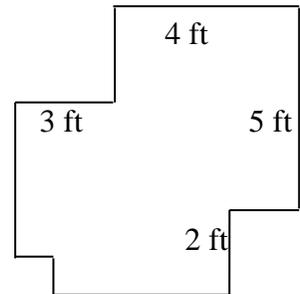
10. If 5 glips equals 2 glups and 3 glups equals 2 glops, then how many glips equal 1 glop? Express your answer as an improper fraction.
11. The average (arithmetic mean) of Huajie's six test scores is 89. What must he score on his seventh test to average a score of 90 on all seven tests?
12. An animal collector just added some new turtles and penguins to his animal collection. He isn't sure how many of each animal he has, though. If the animal collector knows that there are 13 heads and 34 feet total, how many penguins did he add to the collection?
13. Simplify the expression  $\sqrt{420 - \sqrt{420 - \sqrt{420 - \sqrt{420 - \sqrt{420 - \dots}}}}}$  where  $\dots$  indicates the pattern continues indefinitely.
14. How many positive integer multiples of 6 are there less than 1000?
15. Jenny always chooses her clothes and necklaces at random in the morning. She has 5 shirts (1 blue, 1 red, 1 green, 1 yellow and 1 orange). She also has 3 pants (1 red, 1 blue, 1 purple). Lastly, she has 4 necklaces (1 green, 1 white, 1 gold and 1 blue). She selects 1 of each type of item to wear. What is the probability of Jenny choosing 2 blue objects and 1 green object? (Note: she chooses amongst the shirts, then the pants, then the necklaces) Please leave your answer as a simplified fraction.
16. Mrs. Ding must line up her 7 students for a class field trip. However, when putting the students in line, she knows that Bob cannot stand next to Billy, or else they will cause a ruckus. In how many different ways can she line up the children?
17. Hsing wants to sell chocolate bars to buy a new watch that costs \$50 after taxes. He has a box containing large chocolate bars and another box containing small chocolate bars. He makes a profit of \$1.40 for every large chocolate bar he sells and \$0.90 for every small chocolate bar he sells. He does not lose money on unsold, lost or stolen chocolate bars. He manages to sell 40 total chocolate bars before Pepe eats the rest of his supply. Fortunately, he makes just enough money to buy the watch. How many large chocolate bars did he sell?

18. How many ways can the letters in “mathematics” be rearranged if the two “m”s cannot be next to each other and if the word must end in “s”?
19. A palindrome is a number that is read the same backwards as forwards, for instance, 12321. How many more 5-digit palindromes are there than four-digit palindromes? (Note that 01210 is not considered a 5-digit number.)
20. Shirts, pants, and shoes come in three colors with each color coming in three styles. Joy wants a three-part outfit (shirt, pants, shoes), such that each pair of parts (i.e. shirts and pants, shirts and shoes, pants and shoes) has matching colors, matching styles, or both. How many outfits can Akshay choose for Joy?

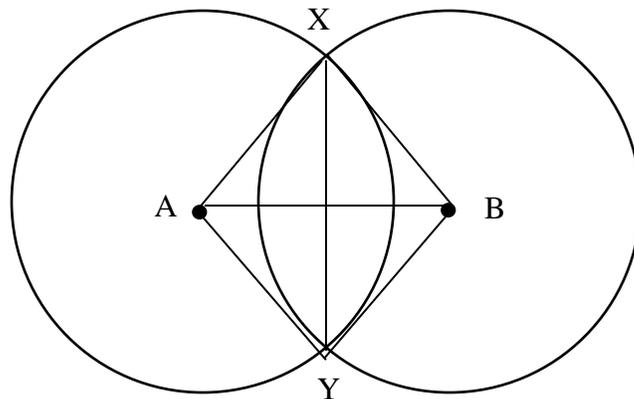
2014 IMSA Junior High Mathematics Competition

8<sup>th</sup> Grade Individual

1. What is 0.0197 rounded to the nearest thousandth?
2. Guy de Maupassant is writing a short story about cafés. If he types at 48 words per minute and his story ends up 1728 words long, how many minutes did it take him to write the story?
3. What is the perimeter of this figure? (Assume all angles are right).



4. What is the sum of the factors of 48?
5. A bag contains 5 blue marbles, 4 red marbles, and 3 yellow marbles. Once you draw a marble you cannot put it back in the bag. What is the probability that you draw a red, yellow, and blue marble in that order?
6. Circles A and B are congruent. Segment AB is 16. X and Y are the intersection points of the two circles. The area of quadrilateral AXBY is 96. Find the length of segment XY.



7. A bag contains blue, red, green, black, and white marbles.  $\frac{1}{3}$  are red,  $\frac{1}{4}$  are blue, and  $\frac{1}{6}$  are black. Of the remaining, 4 marbles are white, and 5 marbles are green. If two marbles are drawn without the replacement of the first, what is the probability that both are blue?
8. How many divisors does 75600 have, including 1 and itself?
9. Two 8-sided dice are rolled. What is the probability that one die lands with 3 facing up and the other lands with 8 facing up? Express your answer as a fraction in simplest form.
10. Pei Pei is taking a very difficult biology course, and his previous exam scores were 95, 94, 89, and 79. What is the lowest score Pei Pei can get on his next test to average an A in the class (90%)?
11. Let  $a + b = 90$  and  $a - b = 40$ . Find  $\frac{a}{b}$  as a simplified fraction.
12. The Little Bitsy Spider lives at the bottom of a rain gutter 10 meters below the roof. Her life's goal is to reach the top of the rain gutter. If every day, she can climb 10 centimeters, and every night, she slips down 3 centimeters, how many days will it take her to reach the roof?
13. Quadrilateral ABCD is a square. One circle is inscribed within ABCD, while another circle circumscribes ABCD. If a point is randomly chosen within the largest circle, what is the probability that it is inside ABCD, but outside the inscribed circle?
14. On Monday, there were 70 apples on display. By the end of the day, some of the apples had been sold. On Tuesday, the manager added an equal number of apples to the apples that were left (he doubled the leftovers). By the end of the day, he had sold the same number of apples as he had sold on Monday. On Wednesday, the manager decided to triple the number of apples that he had left. He sold the same number that day, too. At the end of this day there were no apples left. How many were sold each day?

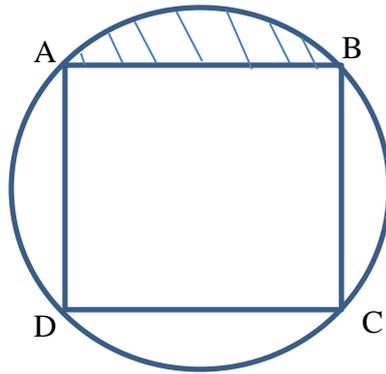
15. Tim is playing in a 7-game chess tournament. He has a 70% chance of winning any given game, a 20% chance of drawing, and 10% chance of losing. What is the probability that he will end the tournament with exactly 5 wins, 1 loss, and 1 draw? Express your answer as a decimal rounded to the nearest thousandth.
  
16. A six-sided die is rolled twice. What is the probability that the sum of the numbers rolled is greater than 8 or that the sum is even? Express your answer as a common fraction.
  
17. A store sells gum in packs of seven sticks and packs of five sticks. Jane wants to buy a certain number of sticks that is possible by buying a combination of seven and five stick packs. What is the largest possible number of sticks that Jane cannot buy?
  
18. You will receive 17 coins and 3 bombs in some order, one at a time. Receiving a bomb destroys all coins in your possession. You may cash out at any time. How many coins can you guarantee to keep?
  
19. The numbers 1 through 10 are arranged along a line. The nine positive differences between consecutive numbers are added together. What is the maximum possible sum?
  
20. What is the remainder when  $4^{64} - 1$  is divided by 3?

2014 IMSA Junior High Mathematics Competition

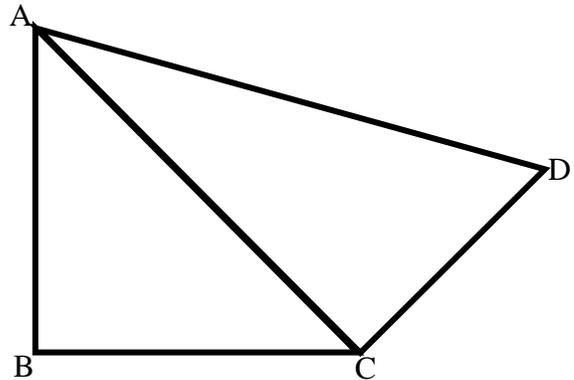
7<sup>th</sup> Grade Team

1. Starting at the origin, Tina rode her horse 3 miles north and then rested for an hour. Later, she went 5 miles east. After turning 270 degrees clockwise, she continued. Which direction is Tina now going?
2. The circumference of a circle is 300 smoots. What is the area (in smoots squared)?
3. How many ways can the number 55 be written as the sum of two prime numbers?
4. If  $a$  and  $b$  are positive integers which sum to 346, what is the minimum value of the product  $a \cdot b$ ?
5. Donny, Jack, and Margaret are siblings. Donny is half Margaret's age. Jack is three times as old as Margaret. Jack is currently 12 years old. How old will Donny be in 10 years?
6. Jacob asks 7 of his friends how many baseball caps each of them owns. His data was as follows: 10, 17, 13, 26, 22, 6, 11. What is the sum of the mean and the median of the data?
7. Danielle is speedily rushing around, checking answers at the speed round of JHMC. It takes her 10 seconds to check a student's answer and 25 seconds to run to the next student. She starts at 0 seconds by checking a student's answer. How many students can she check within 5 minutes?
8. Ronald loves to play with numbers, so he decided to take the mean of 2, 6, 17, 72, 89, and 90. But then he decides that, since his favorite number is 40, he wants to replace the 89 in his list with another number to make the average equal to 40. What number replaced 89?
9. Three names will be drawn one at a time, and the winners each win a prize. Out of the 112 people hoping to win, what is the probability that Sarah will get the First Prize, then Sam will the Second Prize, and that James will not win a prize? Express your answer as a common fraction. Note that a person can win at most one prize.

10. Square ABCD is inscribed a circle. Side BC has length 4 cm. What is the area of the shaded area?



11. If  $\triangle ABC$  and  $\triangle ACD$  are right triangles, and  $\overline{AD} = 10$ , find the sum of the squares of the sides of the quadrilateral ABCD.



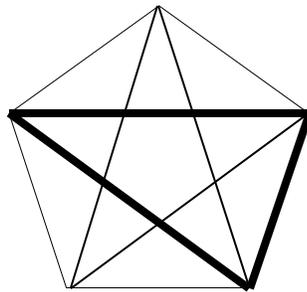
12. What is the units digit of  $1234567^{24}$ ?

13. A certain rectangle with sides of length 3 and  $3\sqrt{3}$  is drawn. A new, similar rectangle is drawn with its longest side having the same length as the diagonal of the original rectangle. The new rectangle now has side lengths 6 and  $2\sqrt{3}$ . This pattern is repeated until the last rectangle has a perimeter greater than 100. How many rectangles are drawn? Include the first and last rectangle in your answer.

14. If  $a$  and  $b$  are numbers and  $a @ b = a^3 - b$ , evaluate  $4 @ (3 @ (2 @ 1))$ .

15. Dawson and Evan are playing a game called Coup. Dawson claims that he has at least one king in his hand, while Evan claims that Dawson doesn't. There are a total of 15 cards in the game, three of which are kings. Dawson has two cards. Evans also has two cards in his hand, and neither of them are kings. What is the probability that Dawson is telling the truth?

16. Malachi has 1000 mini tables, and Hsing and William want to flip them. It takes 40 minutes for Hsing to flip all the tables by himself. It takes 8 minutes for Hsing and William to flip them all together. How long in minutes does it take for William to flip all the tables by himself?
17. A certain student notices a bag of 15 chocolates on his roommate's desk. He's feeling hungry and decides to steal a few, but he won't take more than 3 to avoid suspicions. If each wrapped chocolate is distinct, how many ways can the student steal 3 or fewer chocolates? Clarification: Not stealing any chocolates does not count. He WILL steal something.
18. How many triangles are shown in the figure below? Sides of the triangles may overlap with other lines. One such triangle is outlined for you.



19. Dr. Condie has decided to be extremely nice and generous and has provided his class with donuts! There are 14 students and he has ordered 6 plain donuts, 3 lemon donuts, 4 jelly donuts, and 1 special donut with caviar flavor. Each student receives exactly one donut. How many ways can he distribute the donuts to his class if MoMo, a student, refuses to eat the caviar donut?
20. Aquila and his roommate are having a film marathon. They watch 25 animated short films (never mind how long it took them) and categorize each movie as comedies, fantasies, or romances. A film may be under more than one category, and all films are under at least one category. If they found that 21 films are comedies, 7 are fantasies, and 13 are romances, 2 are comedic fantasies but not romances, 4 are comedic romances but not fantasies, and no films are only romantic fantasies without also being comedies, how many films fall under all three categories?

2014 IMSA Junior High Mathematics Competition

8<sup>th</sup> Grade Team

1. Of 50 students interviewed about the movie *Frozen*, 36 said that they loved the songs. What is the percentage of students who love *Frozen* songs?
2. Rohit wants to save money to buy a new calculus textbook. Suppose he puts \$100 in the Bank of Shah, which pays a compounded interest of 5% each year. How much money will Rohit have after two years?
3. Five tiles labeled 1, 2, 3, 4, 5 are arranged in a line. It is permitted to swap any two tiles that are exactly one tile apart, such as 2 and 4. What is the minimum number of permitted swaps needed to rearrange the tiles into the order 5, 4, 3, 2, 1?
4. Diane wants to buy 2 skirts on sale. One of them has a retail price of \$15, and it is now 30% off. The other skirt has retail price \$12, and it is 10% off. She also has coupon for 20% off the total price she pays. How much does Diane pay if she buys both the skirts?
5. Colleen's two favorite numbers have a sum of 25 and a difference of 13. What is the larger of her two numbers?
6. Ten students take a very, very, very hard test and score an average of 2.3 (out of 1000 points). One student scored an 8, one student scored a 5, and the rest scored either 2 points or 1 point. How many students scored 2 points?
7. Arnold Schoenberg was a musical composer who was also afraid of buildings exceeding a certain height. This peculiar man died on July 13, 1951. If July 13, 1953 was a Monday, which day of the week did Schoenberg die? (Hint: Don't forget to count the leap year.)
8. Felix likes Rubik's cubes. There are 27 little cubes that make up his Rubik's cube, and each of the outside faces of the cube is covered with a sticker. How many little cubes have exactly 2 stickers on them?

9. If  $a \otimes b = \frac{(\text{Greatest Common Factor of } a \text{ and } b)}{(\text{Least Common Multiple of } a \text{ and } b)}$ , what is  $30 \otimes 4 + 15 \otimes 5$  equal to?  
Express your answer as a fraction in simplest form.
10. When an integer is divided by 7, the remainder is 5. What is the remainder if two times that integer is divided by 7?
11. You have a mixture of paint that contains 3 liters of yellow paint and 4 liters of blue paint in Bucket A. Bucket B contains 20 liters of yellow paint. You add some paint from Bucket B into Bucket A to make green paint that is made of yellow and blue paint in the ratio 5:1 (yellow: blue). How many liters of yellow paint remain in Bucket B after creating the green paint?
12. Lawrence and Marie agree to go on a date at 3:00 pm. However, both of them have an unfortunate habit of running late. Marie agrees to wait 15 minutes after her arrival for Lawrence before leaving out of boredom. Lawrence is willing to wait for an hour after his arrival before ditching Marie. The date ends at 5 pm. What is the probability that the two manage to meet up before then?
13. Alice and Bob are coloring some pictures. If it takes 5 minutes for Alice to color 4 pictures alone and 3 minutes for both children to color 4 pictures together, how many complete pictures can Bob color in 15 minutes?
14. If McDonalds sells nuggets in boxes of 3 nuggets and 7 nuggets, what is the greatest number of nuggets Rohit CAN'T buy when he visits McDonalds? (Note that Rohit is the richest person in the world and can afford as many nuggets as he wants.)
15. Define  $a \# b$  as  $(a + b)^2$  for all real numbers  $a$  and  $b$ . For what pair  $(a, b)$  is  $a \# b = 2 \cdot a \cdot b$ ?
16. It's opposite day at IMSA. The '+' sign now signifies conventional subtraction, the '-' signifies conventional addition, the '×' sign signifies conventional division, and the '÷' sign signifies conventional multiplication. Order of operations should remain unchanged. Calculate  $3 \times (5 + 4) \div 12 - 2$ .
17. On the planet Decator every year consists of 12 days. How many Decator citizens must be standing in a room together for them to be greater than 50% sure that at least two of them share the same birthday?

18. Consider the following sequence of letters: MAGIC. If I consider all 120 rearrangements of the letters in the word and arrange these in alphabetical order to form a list, what would the 27<sup>th</sup> entry in the list be?
19. Dawson and Evan are playing a game called Coup. Dawson claims that he has at least one king in his hand, while Evan claims that Dawson doesn't. There are a total of 15 cards in the game, three of which are kings. Dawson has two cards. Evans also has two cards in his hand, and neither of them are kings. What is the probability that Dawson is telling the truth?
20. The SAT has a cooking section, a dinosaur-hunting section, and a trashy-literature section; each section is worth 800 points. Sagar's such a smart kid his score was higher than 2000. Sagar also told us that he is a teenager and his score is equal to his age times 150. What is the sum of all of Sagar's possible SAT scores?